Name:______________________________

MA 3053 Section 01 *Practice Exam 1* *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Let $f : X \to Y$ and $g : Y \to Z$. Prove that if $g \circ f$ is an injection, then f is an injection.

2. Let $f : X \to Y$. Given functions $g, h : W \to X$ such that whenever $f \circ g = f \circ h$, then $g = h$; show that f is injective.

3. Let $f : X \to Y$ and $P_\alpha \subseteq Y$ for every $\alpha \in A$ Show

$$f^{-1}\left(\bigcup_{\alpha \in A} P_\alpha\right) = \bigcup_{\alpha \in A} f^{-1}(P_\alpha)$$

4. Let $f : X \to Y$ and $P_\alpha \subseteq X$ for every $\alpha \in A$ Show

$$f\left(\bigcup_{\alpha \in A} P_\alpha\right) = \bigcup_{\alpha \in A} f(P_\alpha)$$

5. Let f be a real function that is strictly decreasing. Prove that for any $b \in \mathbb{R}$, $f^{-1}(b)$ is either empty or consists of a single element. Deduce from this that f must be an injection.

6. Let f be a real function that is decreasing. Is it possible for f not to be injective? Surjective? Be sure to justify.

7. Let $\sim$ be a relation on $X = \mathbb{Z} \times \mathbb{Z}$ by $(a, b) \sim (c, d)$ if and only if $a + d = b + c$. Which of the properties of reflexivity, symmetry, antisymmetry, and transitivity apply to this relation? If it has a property be sure to prove it, if it doesn't give a counterexample.

8. Let $\mathcal{F}$ be a family of sets and let R be a relation on $\mathcal{F}$ by XRY if and only if $X \subsetneq Y$. Which of the properties of reflexivity, symmetry, antisymmetry, and transitivity apply to this relation? If it has a property be sure to prove it, if it doesn't give a counterexample.